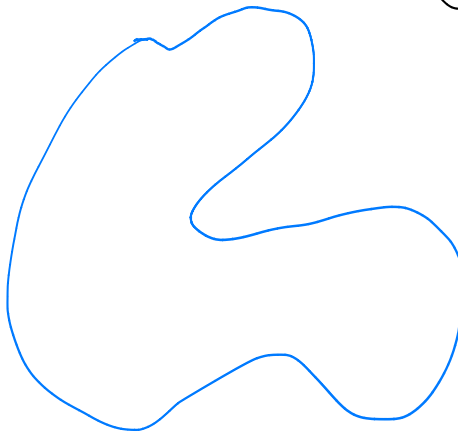
By Sarah Griffith

Lagrange Multipliers



Start with a

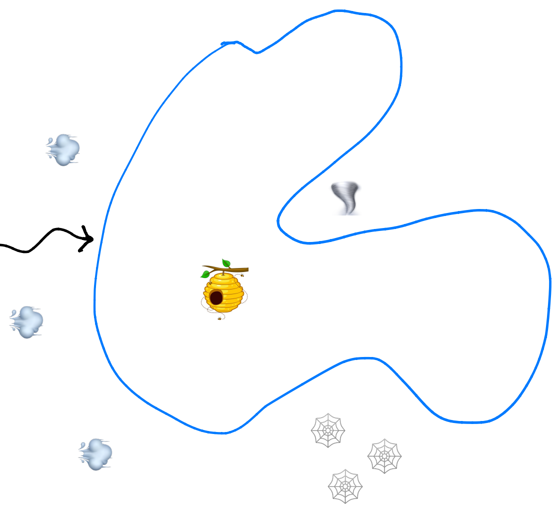
Constraint: $f(x,y)=0$

For example, f could

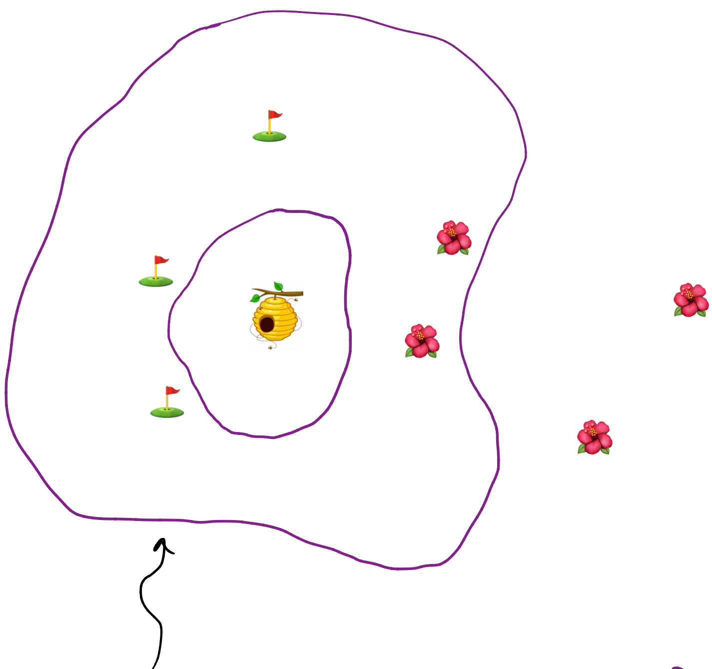
bee:

$$\left(\text{energy cost of flying to and from } (x,y) \text{ to gather pollen}\right) - \left(\text{bee energy for a day}\right)$$

the level set $f(x,y)=0$ (the farthest points a bee could fly to)



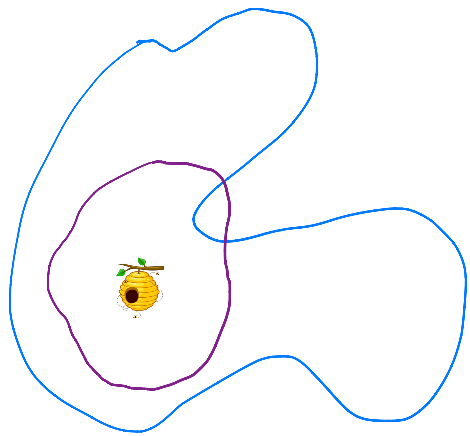
We also have a benefit function $B(x,y)$ that records how much pollen you go by when flying to (x,y)



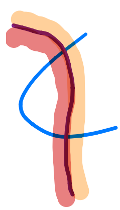
Example level sets for $B(x,y)$

We want to maximize $B(x,y)$ subject to $f(x,y)=0$.

Pick a value k and take $B(x,y)=k$. Can k be the maximum possible benefit on $f(x,y)=0$?



Zoom in:

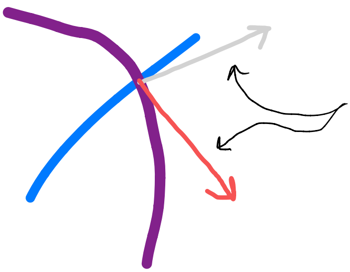


$B(x,y)>k$ on

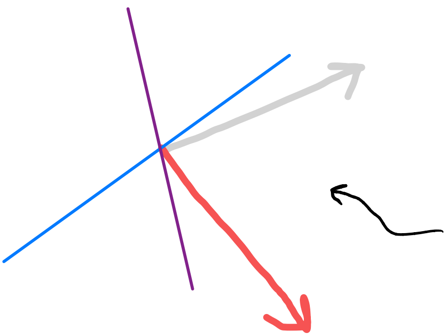
$B(x,y)<k$ on

So the answer is <u>no</u>, because the $f(x,y)=0$ curve crosses over the $B(x,y)=k$ curve at a point of intersection.

More abstractly, $\nabla B(x,y)$ always points perpendicular to level curves, and the same for $\nabla f(x,y)$.



Gradients at intersection point



The curves are well approximated by their tangent lines at 

These cross over each other when the gradients at  are not aligned.

So check all points where ∇f and ∇B are aligned and $f(x,y) = 0$! 🌺🐝